Thermodynamics and Statistical Physics

Part 1– Thermodynamics

Intermediate Exam 3

Tuesday, October 30 2018, 14:00-17:00, Aletta Jacobshal 01

The total number of points that can be reached in this exam is 90.

Final grade = (points/10) + 1.

1) Short questions (**30 pt**)

a) Describe the third law of thermodynamics in your own words. (**2 pt**)

b) Explain why reversible heat engines have the highest efficiency and why efficiency is always smaller than 100%. (**5 pt**)

c) Write down the equation for the statistical definition of the .(**2 pt**) Temperature.

d) The oxidation of iron ("rusting") is described by the reaction

$4\,\mathrm{Fe}\,(solid) + 3\mathrm{O}_2(gas) \rightarrow 2\,\mathrm{Fe}_2\mathrm{O}_3(solid)$, that is accompanied by a **decrease** in entropy $\Delta_r S = -1449.7\ \mathrm{JK}^{-1}$. Explain why this process occurs spontaneously. (**5 pt**)

e) Consider a gas of molecular hydrogen (O_2). We can assume it behaves as a perfect gas. Explain, under which conditions the molar heat capacity equals to $C_V = 5/2\,R$. (**3 pt**)

f) Derive the following relationship: (**3 pt**)

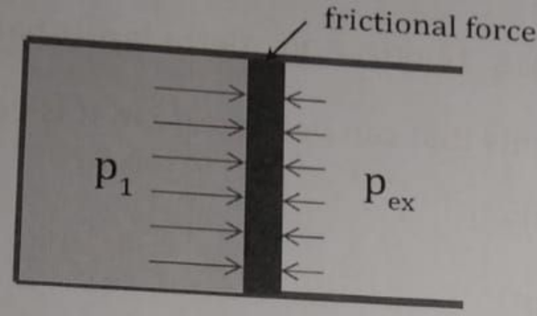
$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V$$

g) Two thermally insulated cylinders, A and B, of equal volume, both equipped with pistons, are connected by a valve. Initially A has its piston fully withdrawn and contains a perfect monoatomic gas at temperature T, while B has its piston fully inserted, and the valve is closed. Calculate the final Temperature of the gas after the valve is fully opened and the gas is slowly drawn into B by pulling out the piston B; piston A remains stationary. The thermal heat capacity of the cylinders can be ignored. (**10 pt**)

2) Expanding piston (**30 pt**)

One mole of perfect gas in a cylinder expands isothermically from volume V_1 to volume V_2. During the expansion, the gas pressure in the cylinder decreases from p_1=200 kPa to p_{ex}=100 kPa. p_{ex} is the external pressure (pressure of the surroundings), i.e. the standard atmospheric pressure outside the cylinder. The system is always in thermal equilibrium with the surroundings at T=300 K.

The expansion proceeds by movement of a piston (see figure). A frictional force between the piston and the cylinder almost compensates the force due to the (decreasing) pressure in the cylinder. The piston thus moves slowly, without acceleration. Cylinder and piston are perfect heat conductors.



a) In this expansion process, due to the frictional force, the net force that acts on the piston is very small throughout the entire process. However, because the additional force is a friction force, the expansion in NOT reversible. Explain. **(4 pt)**
b) Calculate the expansion work of the gas in the cylinder against the constant atmospheric pressure of the surroundings. **(8 pt)**
c) What is the total change in internal energy (ΔU) of the (perfect!) gas in the cylinder during this isothermal process. Explain! **(3 pt)**
d) Use the results from a) and b) together with the thermodynamic definition of entropy, to calculate the entropy change of the surroundings. **(8 pt)**
e) Calculate the total entropy change of the system and the surroundings. **(7 pt)**

3) A thermodynamic cycle (15 pt)

A heat engine is operating between two heat reservoirs 1 and 2, that have constant temperatures of T_1= 450 K and T_2 (with T_2 unknown). The entire system (the two reservoirs and the engine) is isolated. The efficiency of the engine is 20% (ε=0.2). During the cycle, 80 J of work is delivered by the engine. The increase in total entropy ΔS_{tot} after one cycle is 0.25 J/K.

a) Assume $T_2<T_1$. Explain why q_1=400 J, with q_1 the amount of heat extracted from reservoir 1, and why q_2=320 J, with q_2 the amount of heat deposited in reservoir 2. **(5 pt)**
b) Assume $T_2>T_1$. Calculate q_1 and q_2 for this situation. Explain explicitly whether heat is being deposited or extracted into or from each of the reservoirs. **(5 pt)**
c) Calculate the temperature T_2 (hint: use the total entropy). Assume that the two heat reservoirs have a very large, constant volume. Provide solutions for both situations $T_2<T_1$ and $T_2>T_1$. **(5 pt)**

4) Heat flow between two cylinders (15 pt)